S5-07

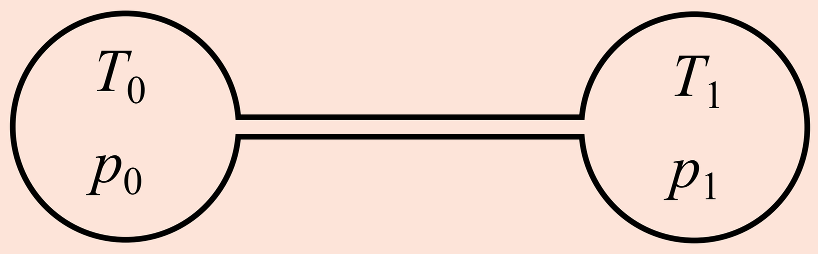
Effect of Kinetic Boundary Condition on the Thermal Transpiration Coefficient

Hiroshi SUGIMOTO and Kenjiro AMAKAWA

Department of Aeronautics and Astronautics, Kyoto University, JAPAN

PROBLEM

We numerically investigate the free molecular flow in a tube that connects two reservoirs with different temperatures T_0 and T_1. This is well known example of the *thermal transpiration* of rarefied gas; The thermal transpiration flow is induced in the direction from the colder reservoir to hotter one, and this induces the pressure difference between two reservoirs. At the steady state with zero mass flow through the tube, the pressure ratio p_1/p_0 is determined by the temperature ratio T_1/T_0:



$$\frac{p_1}{p_0} = \left(\frac{T_1}{T_0}\right)^{\gamma} \qquad (\gamma:\ \text{free molecular thermal transpiration coefficient}).$$

Analysis with Maxwell model of boundary condition on the tube surface [1] leads $\gamma \equiv 1/2$. On the other hand, the experiments [2, 3] shows that $\gamma < 1/2$ for long tubes. Several authors showed that $\gamma < 1/2$ is possible by modern boundary conditions, e.g., Cercignani-Lampis boundary condition. [4–6] In the present study, we try simple model boundary condition to investigate what physical property of gas-wall interaction gives large effect on the value of γ.

BACKGROUND

Theorem (Maxwell B.C.)

- $\gamma = 1/2$ irrespective of the channel shape and distributions of surface temperature & accommodation coefficient α [1]
- α does not depend on molecular velocity $\boldsymbol{\xi}$
  - Maxwell [7]: α for grazing molecules may be small

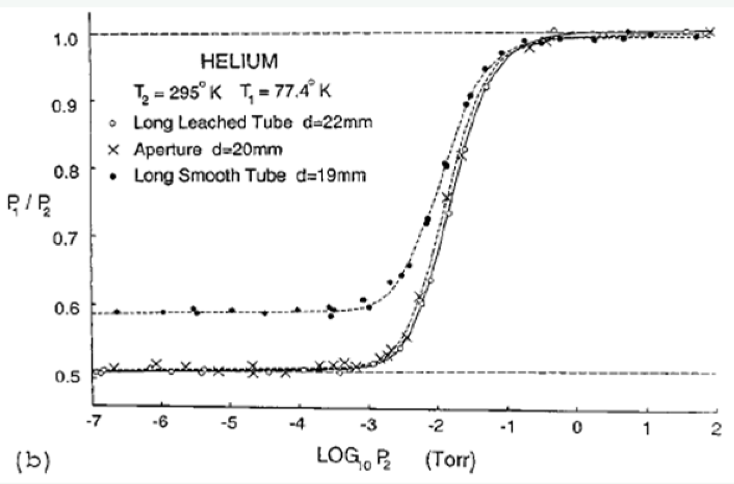
Modern B. C.: $\gamma \neq 1/2$!

- $\gamma \neq 1/2$ is possible for, e.g., Cercignani-Lampis and extended Maxwell model [5, 6] or Epstein model [4]

Question: What physical property (molecular motion) is important ?

Experiment [2, 3]

- Helium & long glass tube: $\gamma \approx 0.4$ [2]

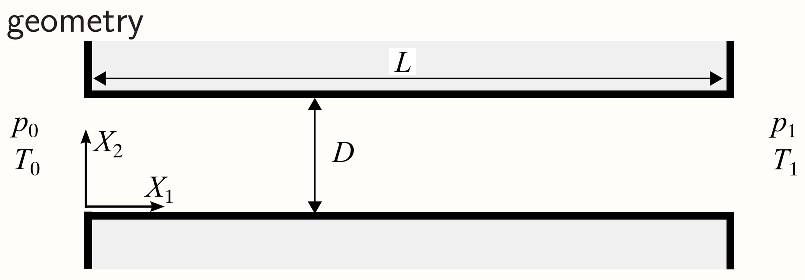


Application of $\gamma \neq 1/2$

- Accommodation pump [8]
- 28 stage pump marks pressure ratio 23

ANALYSIS

- 2D geometry



- Basic equation

$$\xi_1 \frac{\partial f}{\partial X_1} + \xi_2 \frac{\partial f}{\partial X_2} = 0 \qquad (1)$$

- B.C. at reservoirs ($* = 0$ for $X_1 = 0$; $* = 1$ for $X_1 = L$)
  Maxwellian at rest with $\rho_* (= p_*/RT)$, T_*:

$$f = \sigma_* K(|\boldsymbol{\xi}|, \beta_*), \quad K(\xi, \beta) = \frac{1}{\beta^2}\exp\left(-\frac{\xi^2}{\beta}\right),$$

$$\sigma_* = \frac{\rho_* \beta_*^{1/2}}{\pi^{3/2}}, \qquad \beta_* = 2RT_*, \qquad (2)$$

- B.C. at wall: Maxwell model with variable α
  $\alpha(\boldsymbol{\xi})$: function of the speed of incoming molecules $\boldsymbol{\xi}$ [9]:

$$f(\xi_n > 0) = \alpha(\boldsymbol{\xi} - 2\xi_n \boldsymbol{n})\, \sigma_W K(|\boldsymbol{\xi}|, \beta_W) + \left[1 - \alpha(\boldsymbol{\xi} - 2\xi_n \boldsymbol{n})\right] f(\boldsymbol{\xi} - 2\xi_n \boldsymbol{n}), \qquad (3)$$

where $\xi_n = \boldsymbol{\xi}\cdot\boldsymbol{n}$ and σ_W is determined by mass conservation:

$$\int_{|\xi|<\infty} \xi_n f \, d\boldsymbol{\xi} = 0. \qquad (4)$$

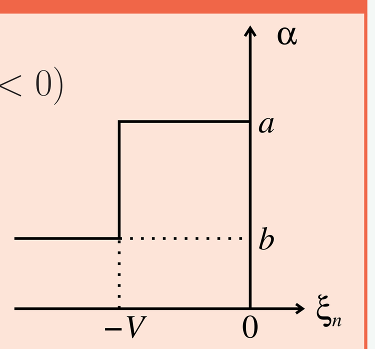
$\beta_W = 2RT_W$, T_W: wall temperature, $\boldsymbol{n}$: unit normal pointing to the gas.

Present work: $T_W = T_0(1 - X_1/L) + T_1 X_1/L$.

Function $\alpha(\boldsymbol{\xi})$

Model-A

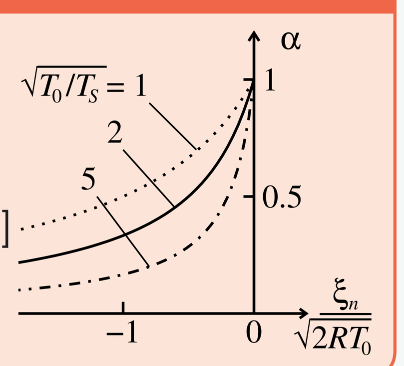
$$\alpha(\boldsymbol{\xi}) = \begin{cases} a & (-V \le \xi_n < 0) \\ b & (\xi_n < -V) \end{cases}$$



- Artificial & simple

Model-B

$$\alpha(\boldsymbol{\xi}) = \frac{1}{1 - \xi_n/\sqrt{2RT_S}}$$

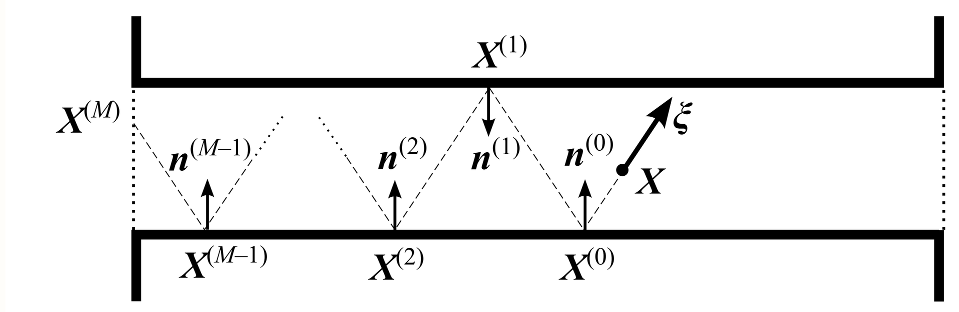


- Theory: Borman et.al [10]
- Consistent with experiment [11]

α does not depend on $\boldsymbol{X}$ or T_W in this work

- Method of solution
  - Backtracking the path of specular reflecting molecule leads [1]

$$f(\boldsymbol{X}, \boldsymbol{\xi}) = \sum_{i=0}^{M-1} \left[1 - \alpha(-|\xi_2|)\right]^i \alpha(-|\xi_2|)\, \sigma_W^{(i)} K\left(|\boldsymbol{\xi}|, \beta_W^{(i)}\right) + \left[1 - \alpha(-|\xi_2|)\right]^M \sigma_W^{(M)} K\left(|\boldsymbol{\xi}|, \beta_W^{(M)}\right) \quad (5)$$



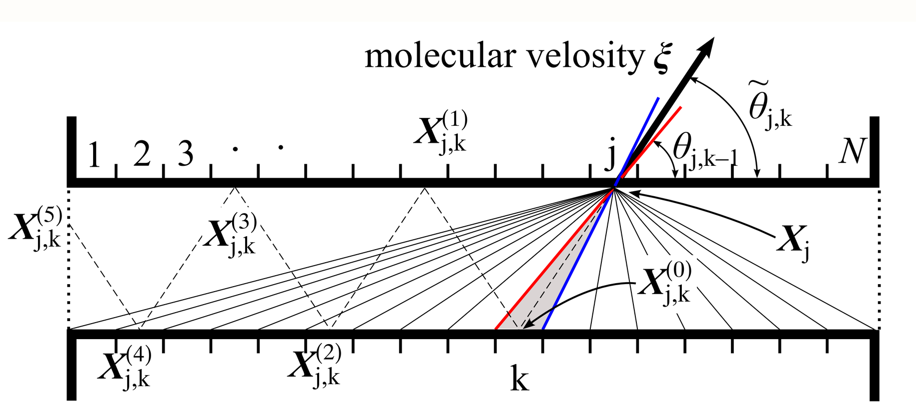
Series $\boldsymbol{X}^{(i)}$ is determined by $\boldsymbol{X}$ and $\boldsymbol{\xi}$.
$\beta_W^{(i)} = \beta_W\left(\boldsymbol{X}^{(i)}\right)$, $\sigma_W^{(i)} = \sigma_W\left(\boldsymbol{X}^{(i)}\right)$.
$\beta_W^{(M)}$ and $\sigma^{(M)}$ are determined by B. C. at reservoir

  - The distribution of $\sigma_W(\boldsymbol{X})$ is determined by (4); Equation (4) is rewritten as

$$\sigma_W(\boldsymbol{X}) = \int_{\xi_n<0} \xi_n \alpha(-|\xi_2|)\, f(\boldsymbol{X}, \boldsymbol{\xi})\, d\boldsymbol{\xi} \Big/ \int_{\xi_n<0} \xi_n \alpha(-|\xi_2|)\, K(|\boldsymbol{\xi}|, \beta_W(\boldsymbol{X}))\, d\boldsymbol{\xi}. \qquad (6)$$

Substituting (5) into (6) leads the integral equation for $\sigma_W(\boldsymbol{X})$.

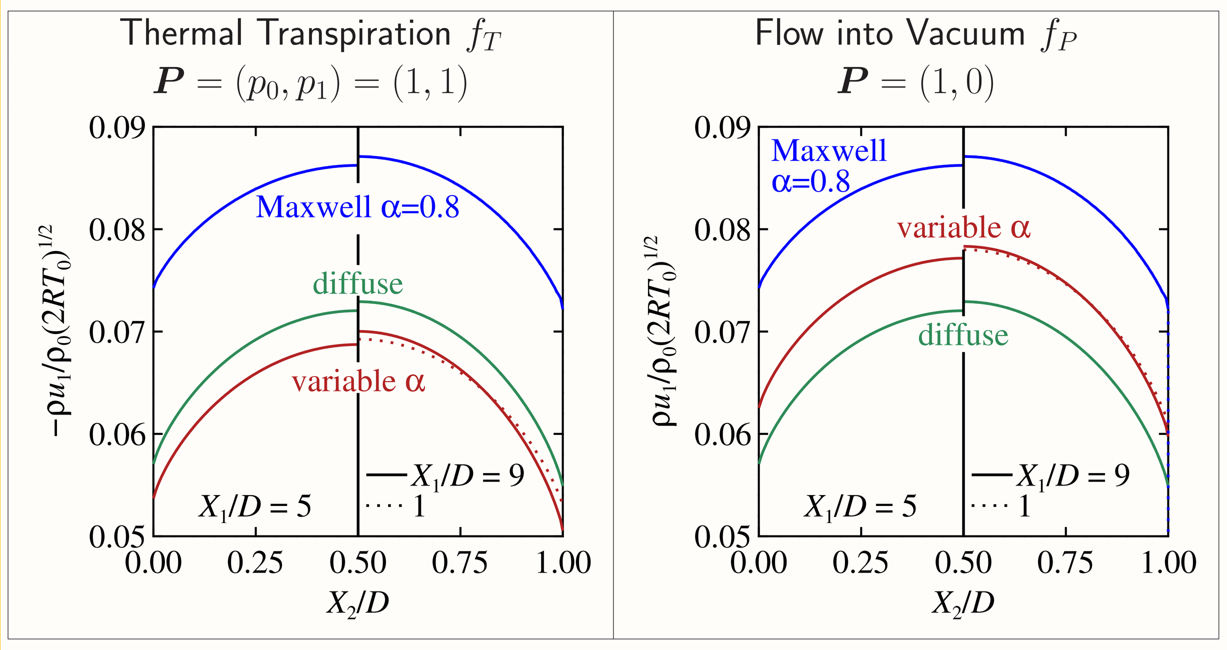
- Numerical Calculation
  - Cylindrical coordinate (ξ, θ, ξ_z) for the integral in Eq. (6).
  - Divide wall into N uniform segments. The integral over θ is carried out using the segments:



- Consider σ_{Wj} at center of segments $\boldsymbol{X}_j$ $(1 \le j \le N)$.
- Integrand for σ_{Wj} from the opposite wall is evaluated at $\tilde{\theta}_{j,k}$ $(1 \le k \le N)$, the angle determined by the centers of segments j and k.
- Solve linear equations for σ_{Wj}

RESULTS

Basic solutions for a given temperature distribution T_W



$L/D = 10$, $T_1/T_0 = 0.25$, calculation with $N = 2000$.
Variable α: Model-A
($a = 1$, $b = 0.8$, $V = 0.5\sqrt{2RT_0}$)

Solution for $\boldsymbol{P} = (p_0, p_1)$:

$$f = p_1\left[f_T + \left(\frac{p_0}{p_1} - 1\right) f_P\right]$$

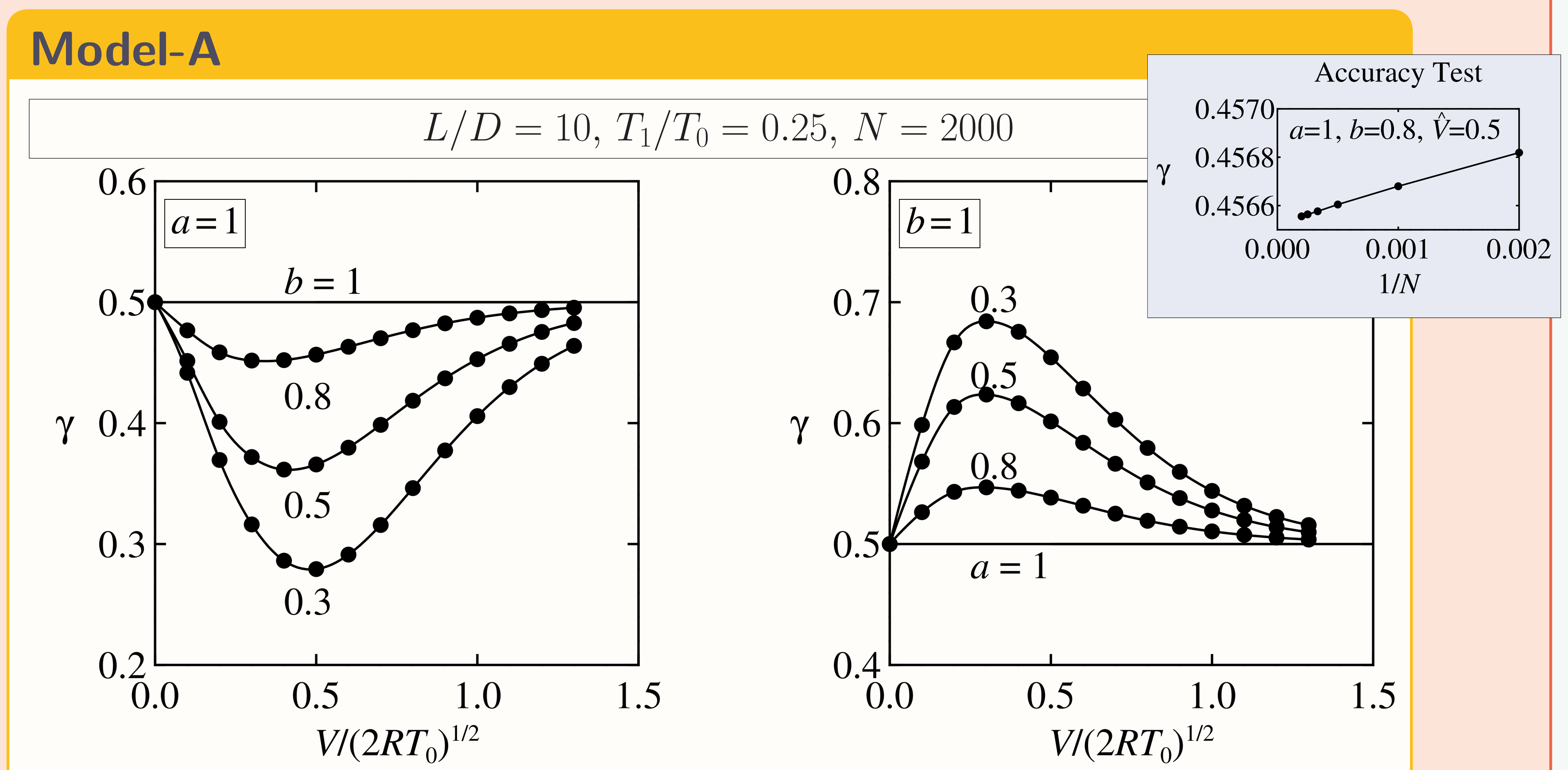
The thermal transpiration coefficient is obtained by

$$\int_0^D \int \xi_1 \, f \, d\boldsymbol{\xi} dX_2 = 0$$

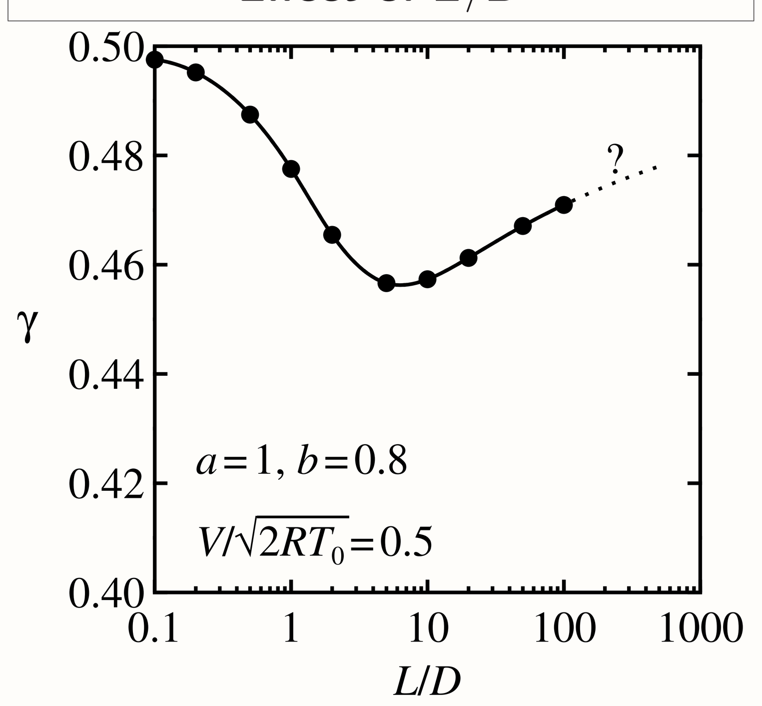
Thermal transpiration coefficient γ

Model-A

$L/D = 10$, $T_1/T_0 = 0.25$, $N = 2000$



Effect of L/D

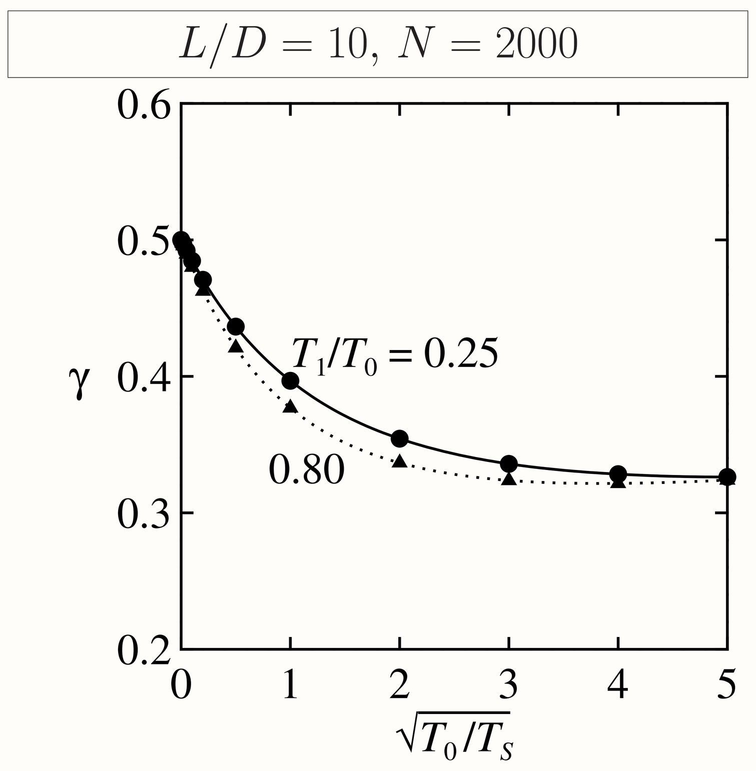


Local flow velocity $|\boldsymbol{u}(\boldsymbol{X})|/\sqrt{2RT_0} \lesssim 10^{-6}$

Diffuse reflection for small $|\xi_n|$
$\Longrightarrow \gamma < 1/2$

Model-B

$L/D = 10$, $N = 2000$



The value of γ slightly depends on temperature ratio of reservoirs

CONCLUSION

- Experimental result with $\gamma < 1/2$ is well described by the original Maxwell model of kinetic boundary condition.
- Velocity dependence of accommodation coefficient is important for the value of γ.
- Diffuse reflection for grazing molecules $\Longrightarrow \gamma < 1/2$.
- γ depends on channel length, temperature ratio etc.

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sugimoto@kuaero.kyoto-u.ac.jp